CS772: Deep Learning for Natural Language Processing (DL-NLP)

Feedforward network and backpropagation

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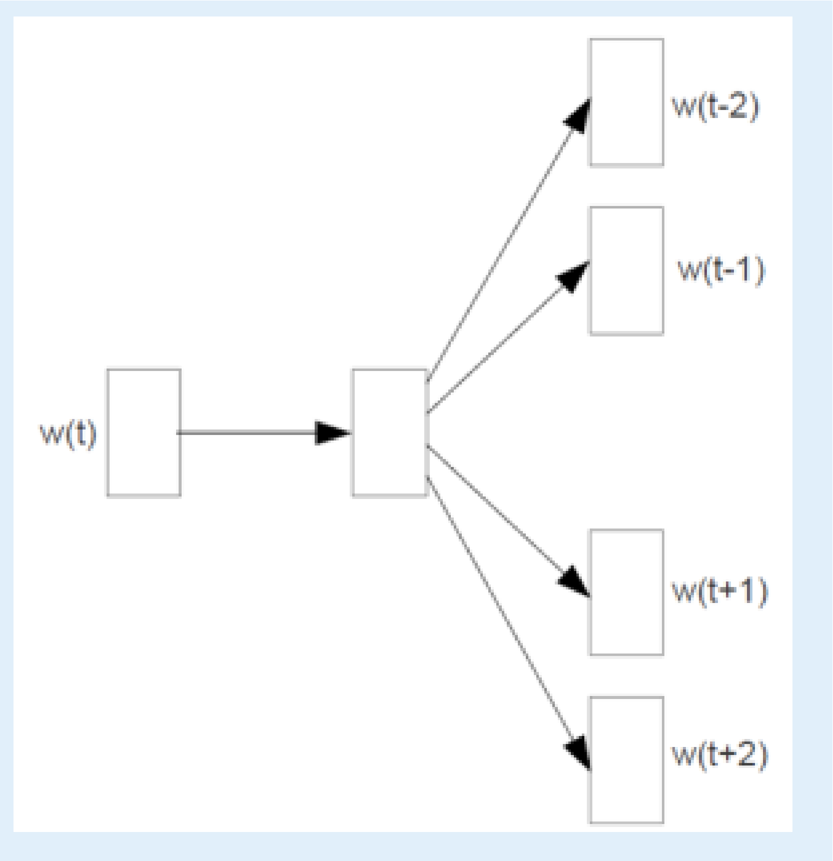
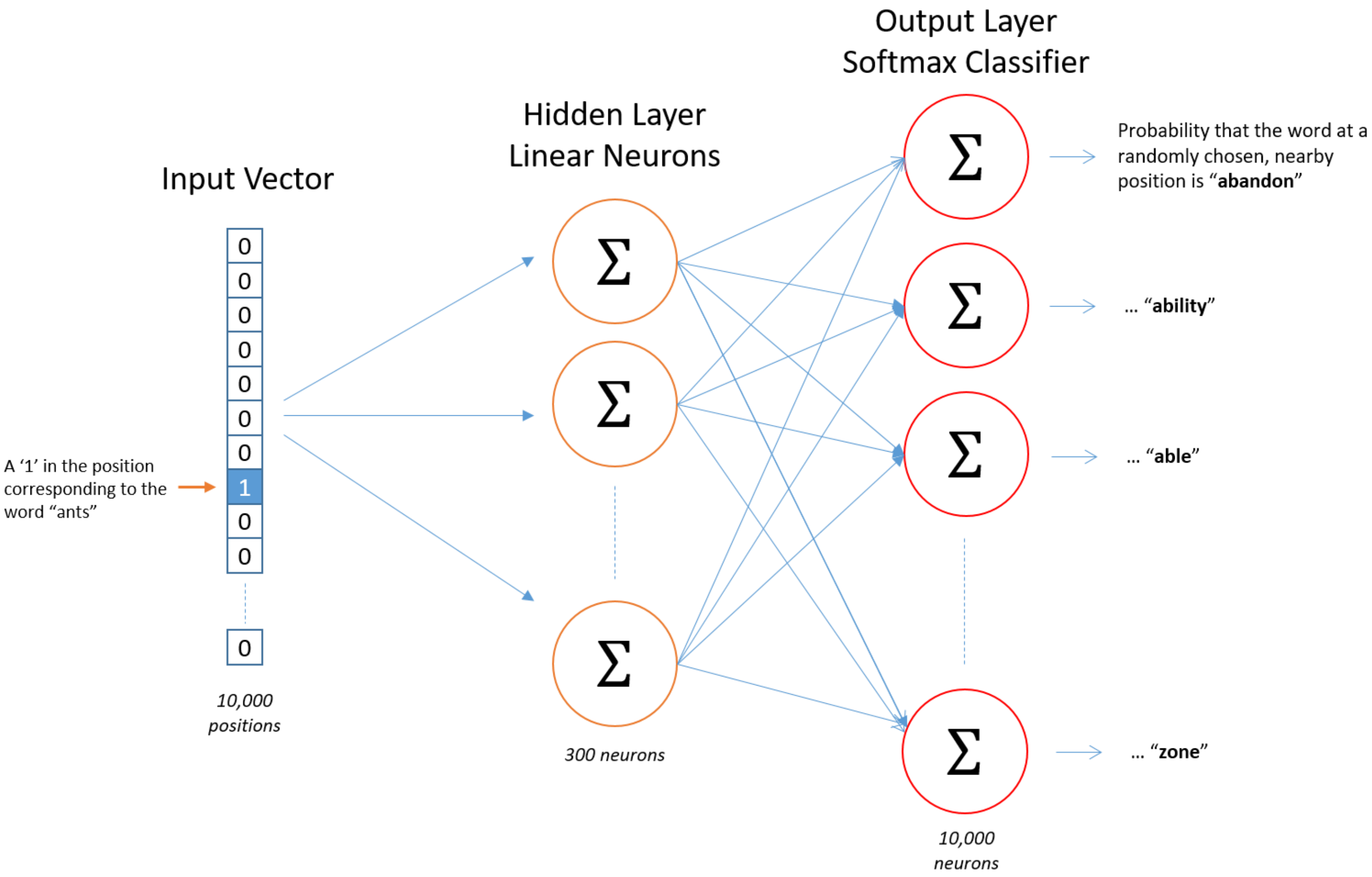
Skip gram- an example of FFNN



For CBOW:

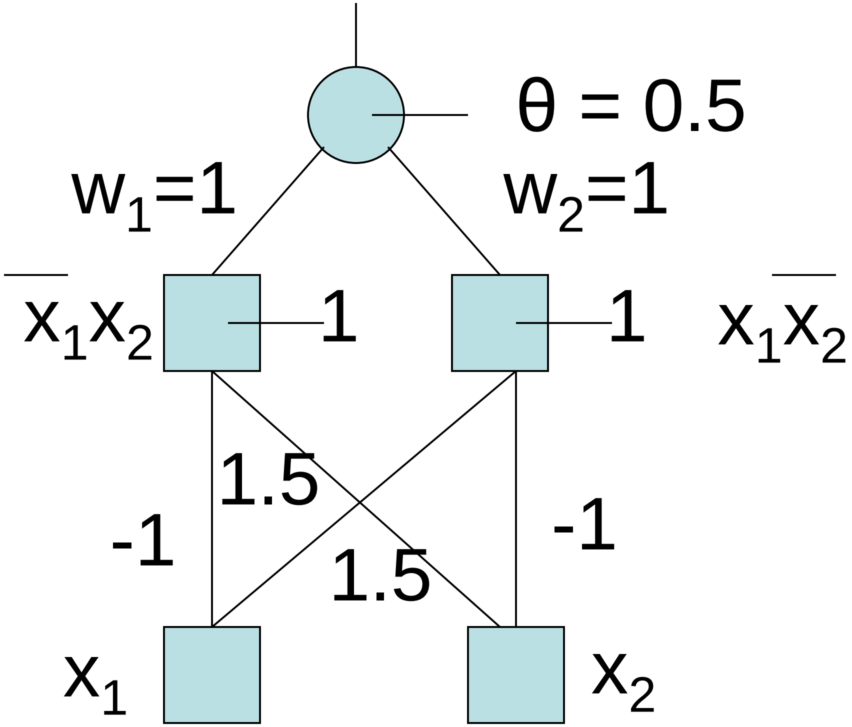
Just reverse the
Input-Ouput

Skip Gram: more details

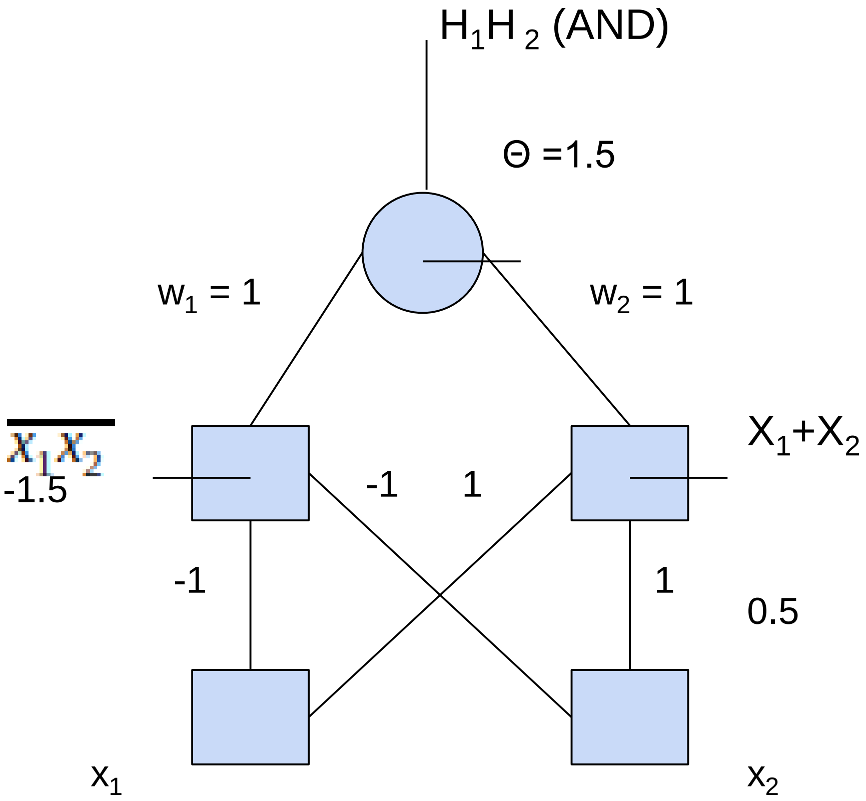

Input Vector
0
0
0
0
0
0
0
1
0
0
0
10,000 positions
A '1' in the position corresponding to the word "ants"
Hidden Layer Linear Neurons
Σ
Σ
Σ
300 neurons
Output Layer Softmax Classifier
Σ
Σ
Σ
Σ
10,000 neurons
Probability that the word at a randomly chosen, nearby position is "**abandon**"
... "**ability**"
... "**able**"
... "**zone**"

Feedforward Network and Backpropagation

Example - XOR



Alternative network for XOR

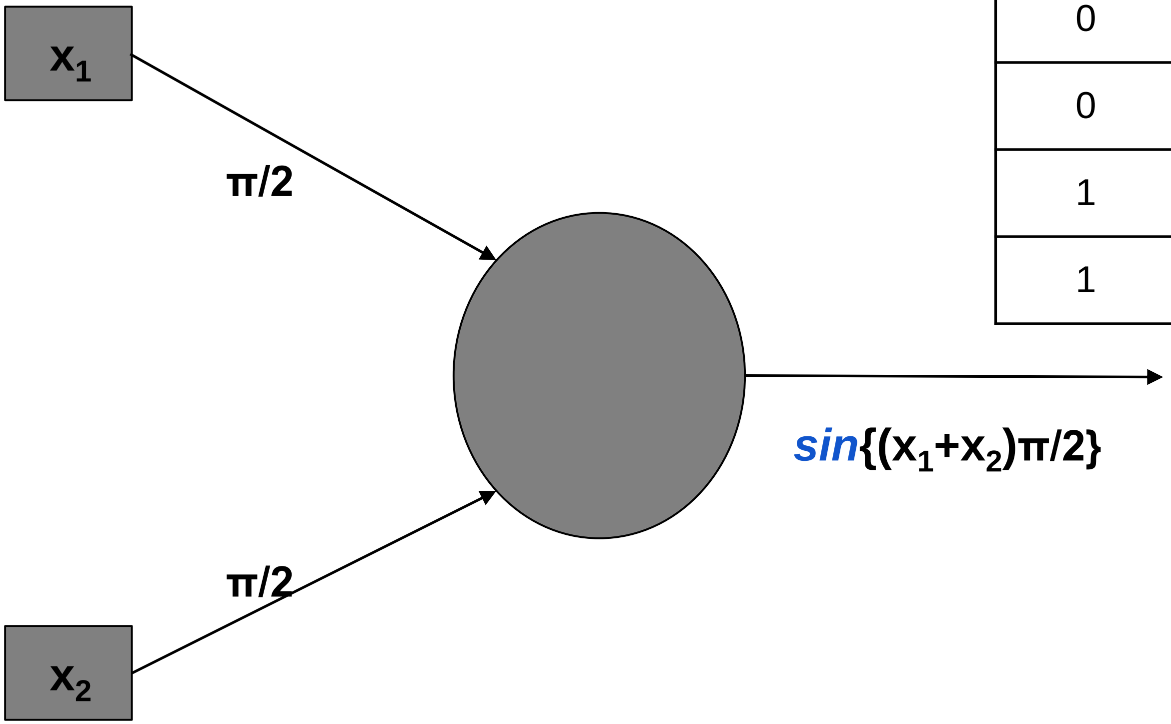


- XOR: not possible using a single perceptron
- Hidden layer gives more computational capability
- Deep neural network: With multiple hidden layers
- Kolmogorov’s theorem of equivalence proves equivalence of multiple layer neural network to a single layer neural network, and each neuron have to correspond to an appropriate functions.

Compositionality

- XOR being computed as OR(X_1'X_2, X_1X_2') or as AND((X_1'+X_2'),(X1+X2)) is an example of a nonlinearly separable function computed as composition of linearly separable functions)
- In general not possible for most practical situations like weather prediction, stock market prediction etc.

XOR neuron with sin()



x_1	x_2	Output
0	0	0
0	1	1
1	0	1
1	1	0

Question

- Since SINE can compute XOR, why do not we use sine neuron for practical applications?

Exercise: Back-propagation

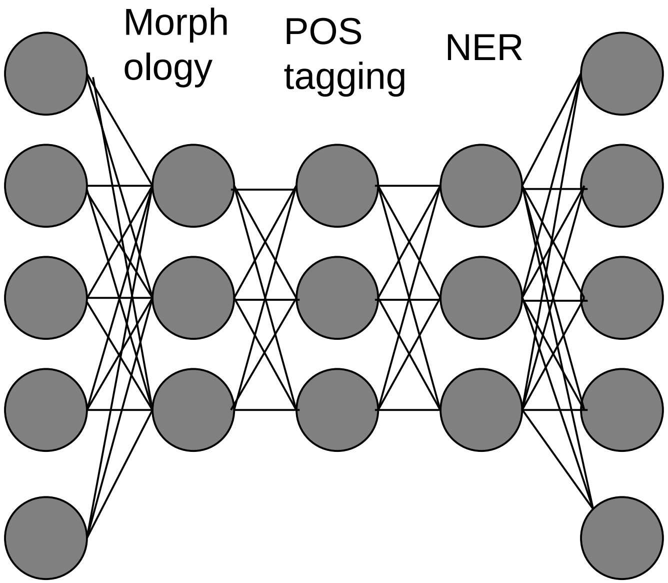
- Implement back-propagation for XOR network

- Observe
  - Check if it converges (error falls below a limit)

  - What is being done at the hidden layer

What a neural network can represent in NLP: Indicative diagram

- Each layer of the neural network possibly represents different NLP stages!!



Batch learning versus Incremental learning

- Batch learning is updating the parameters after ONE PASS over the whole dataset
- Incremental learning updates parameters after seeing each PATTERN (input-ouput pair)
- An epoch is ONE PASS over the entire dataset
  - Take XOR: data set is V_1=(<0,0>, 0), V_2=(<0,1>, 1), V_3=(<1,0>, 1), V_4=(<1,1>, 0)
  - If the weight values are changed after each of Vi, then this is incremental learning
  - If the weight values are changed after one pass over all V_is, then it is bathc learning

Can we use PTA for training FFN?

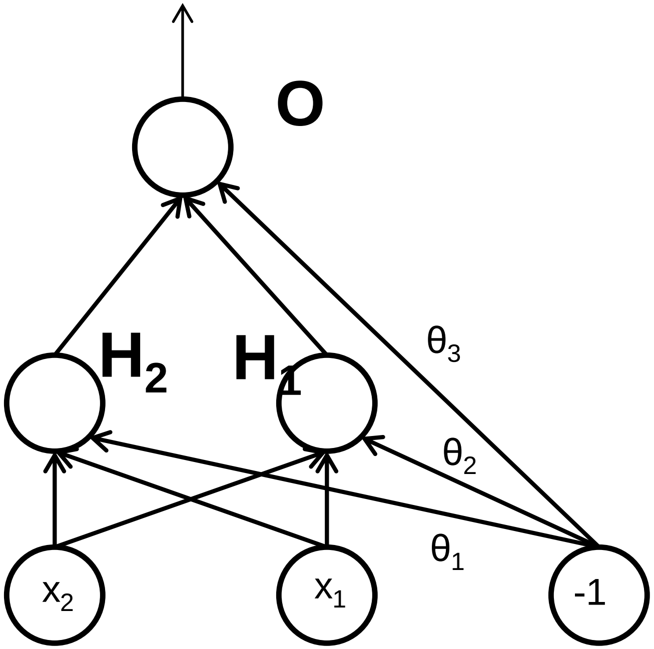
0, 0	0
0, 1	1
1, 0	1
1, 1	0

⇨

-1, 0, 0	0
-1, 0, 1	1
-1, 1, 0	1
-1, 1, 1	0

⇨

1, 0, 0	0
-1, 0, 1	1
-1, 1, 0	1
1, -1, -1	0



No, else the individual neurons are solving XOR, which is impossible.

Also, for the hidden layer neurons we do nothave the i/o behaviour.

Note: This n/w is NOT a pure FFNN; there is jumping of lair.

Gradient Descent Technique

- Let E be the error at the output layer
- *i* goes over *N* neurons in the o/p layer, *j* goes over *P* patterns

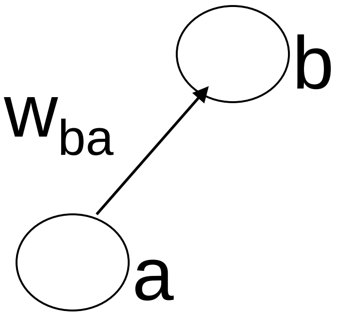
$$E = \frac{1}{2} \sum_{j=1}^{P} \sum_{i=1}^{N} (t_i - o_i)_j^2$$

- t_i = target output; o_i = observed output

- E.g.: XOR:– *P=4 and N=1*

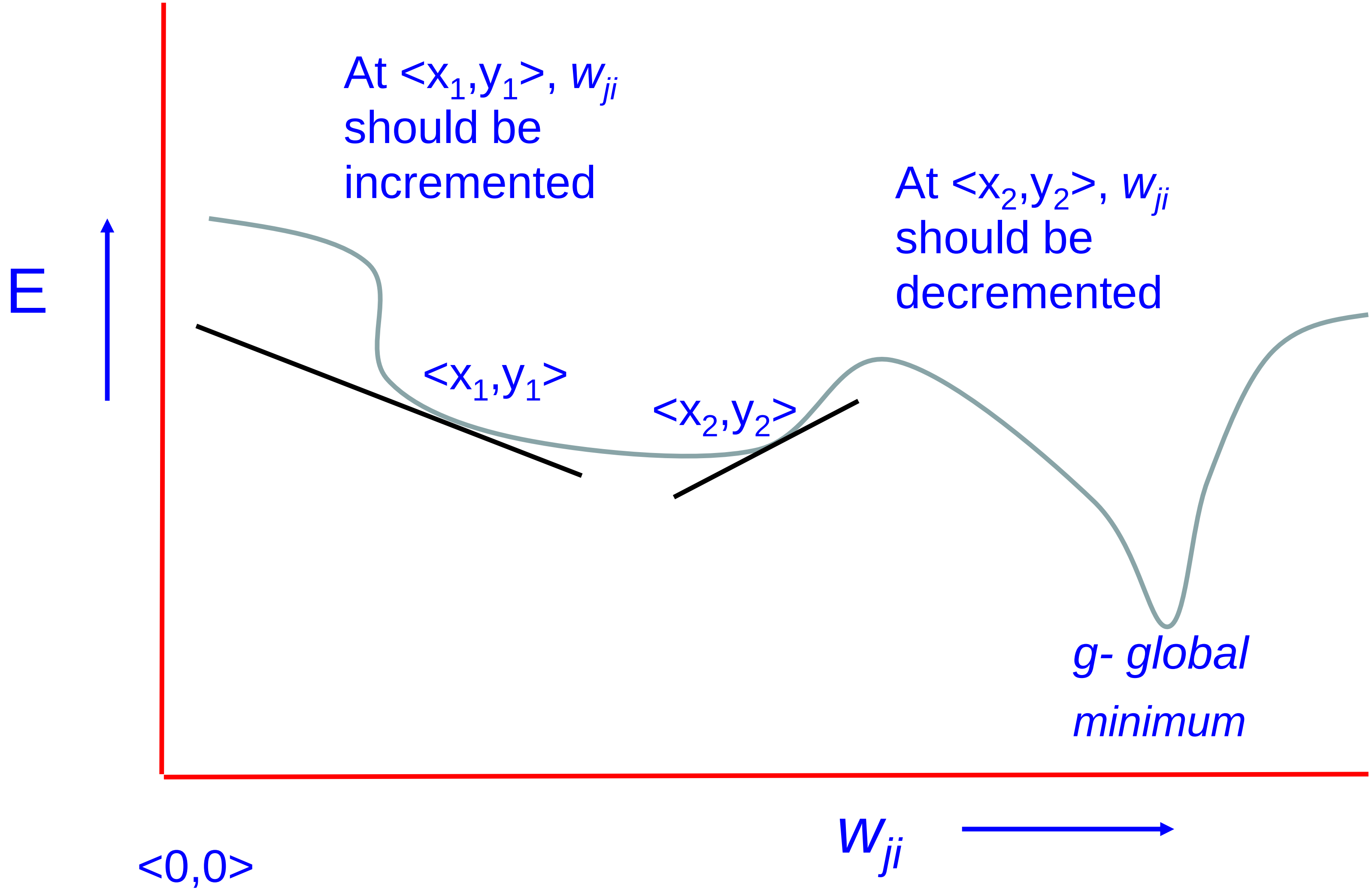
Weights in a FF NN

- w_{ba} is the weight of the connection from the a^{th} neuron to the b^{th} neuron
- E *vs* $\overline{W}$ surface is a complex surface in the space defined by the weights w_{ij}
- $-\frac{\delta E}{\delta w_{ba}}$ gives the direction in which a movement of the operating point in the w_{mn} co-ordinate space will result in maximum decrease in error



$$\Delta w_{ba} \propto -\frac{\delta E}{\delta w_{ba}}$$

Intuition for gradient descent



At $<x_1,y_1>$, w_{ji} should be incremented

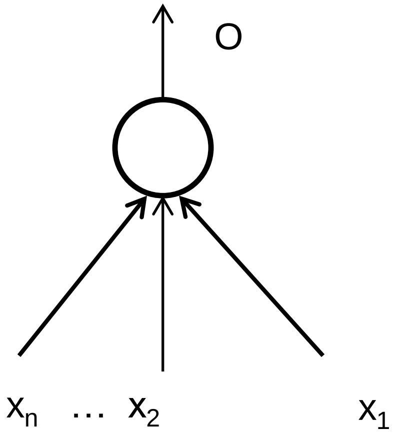
At $<x_2,y_2>$, w_{ji} should be decremented

g- global minimum

Pertains to life!!

- Gradient descent in greedy in nature, E ALWAYS decreases
- Can get stuck in local minimum, miss global minimum
- So: “greed does not always pay”, “short term gains may not lead to long term gains”, “local optimizations need not always lead to global optimizations”

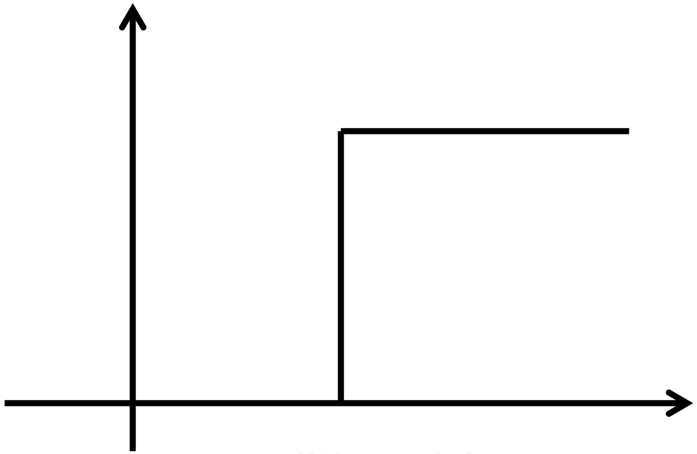
Step function v/s Sigmoid function



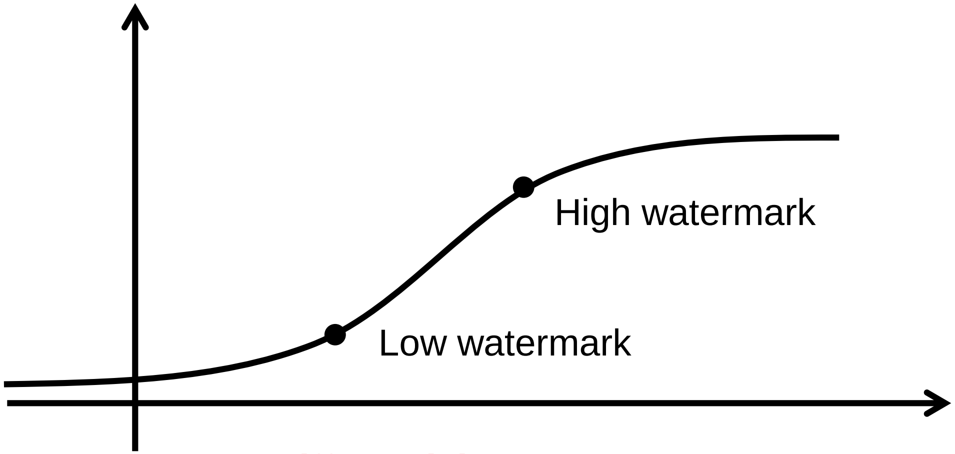
$$
\begin{aligned}
O &= f(\sum w_i x_i) \\
  &= f(net)
\end{aligned}
$$

So partial derivative of O w.r.t. net is

$$\frac{\delta O}{\delta net}$$



Non-differentiable



Differentiable

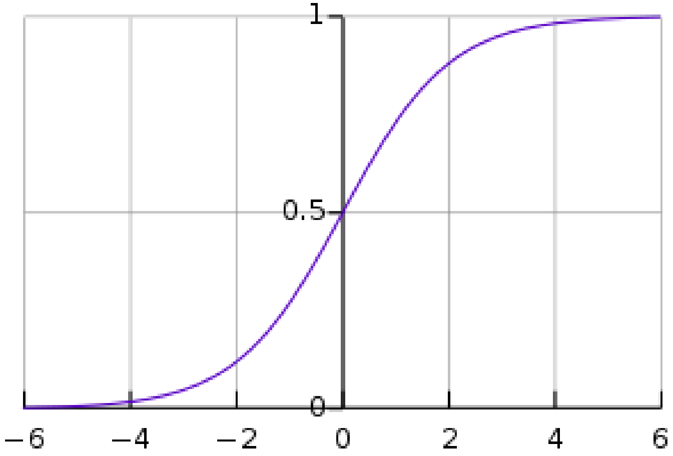
Sigmoid function

$$y = \frac{1}{1+e^{-x}}$$

$y = \frac{1}{1+e}$

$$\frac{dy}{dx} = y(1-y)$$

Sigmoid function



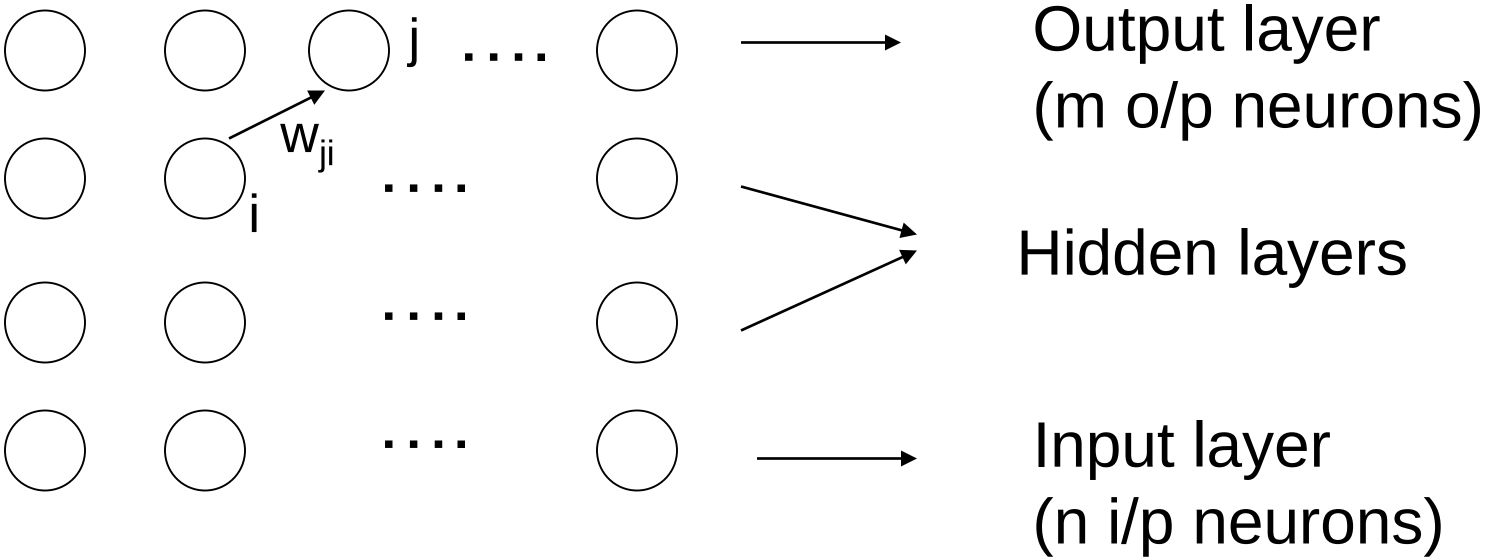
$$f(x) = \frac{1}{1+e^{-x}}$$

$$\begin{aligned}
f(x) &= \frac{1}{1+e^{-x}} \\
\frac{df(x)}{dx} &= \frac{d}{dx}\left(\frac{1}{1+e^{-x}}\right) \\
&= \frac{e^{-x}}{(1+e^{-x})^{-2}} \\
&= \frac{1}{1+e^{-x}}\left(1 - \frac{1}{1+e^{-x}}\right) \\
&= f(x).(1-f(x))
\end{aligned}$$

Interesting point

- Biological (neurophysical) plausibility of sigmoid function
- The saturating behaviour of sigmoid neuron for very large signals (derivative →0) is said to be a "saviour" for the brain
- Intense emotions (joy, sorrow, anger) produce large signals in brain neurons which through positive feedback can lead to brain damage (haemorrhage)
- Saturation avoids this danger

Backpropagation algorithm



Output layer
(m o/p neurons)

Hidden layers

Input layer
(n i/p neurons)

- Fully connected feed forward network
- Pure FF network (no jumping of connections over layers)

Gradient Descent Equations

$$\Delta w_{ji} = -\eta \frac{\delta E}{\delta w_{ji}} (\eta = \text{learning rate}, 0 \le \eta \le 1)$$

$$\frac{\delta E}{\delta w_{ji}} = \frac{\delta E}{\delta net_j} \times \frac{\delta net_j}{\delta w_{ji}} (net_j = \text{input at the j}^{th} \text{ neuron})$$

$$\boxed{\frac{\delta E}{\delta net_j} = -\delta j}$$

$$\Delta w_{ji} = \eta \delta j \frac{\delta net_j}{\delta w_{ji}} = \eta \delta j o_i$$

A quantity of great importance

Backpropagation – for outermost layer

$$\delta j = -\frac{\delta E}{\delta net_j} = -\frac{\delta E}{\delta o_j} \times \frac{\delta o_j}{\delta net_j} (net_j = \text{input at the j}^{th} \text{ layer})$$

$$E = \frac{1}{2}\sum_{i=1}^{N}(t_j - o_j)^2$$

$$\text{Hence, } \delta j = -(-(t_j - o_j)o_j(1 - o_j))$$

$$\Delta w_{ji} = \eta(t_j - o_j)o_j(1 - o_j)o_i$$

Observations from Δw_{ji}

$$\Delta w_{ji} = \eta(t_j - o_j)o_j(1 - o_j)o_i$$

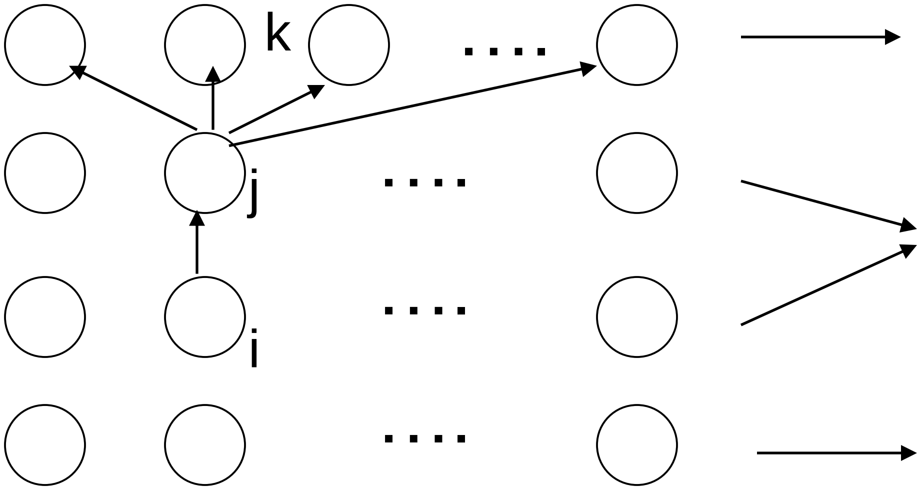
$$\Delta w_{ji} \to 0 \quad \text{if,}$$

$$1.\ o_j \to t_j \quad \text{and/or}$$

$$\left.\begin{array}{l} 2.\ o_j \to 1 \quad \text{and/or} \\ 3.\ o_j \to 0 \quad \text{and/or} \end{array}\right\} \text{Saturation behaviour}$$

$$\left.4.\ o_i \to 0 \qquad\right\} \text{Credit/Blame assignment}$$

Backpropagation for hidden layers



Output layer
(m o/p neurons)

Hidden layers

Input layer
(n i/p neurons)

δ_k is propagated backwards to find value of δ_j

Backpropagation – for hidden layers

$$\Delta w_{ji} = \eta \delta j o_i$$

$$\delta j = -\frac{\delta E}{\delta net_j} = -\frac{\delta E}{\delta o_j} \times \frac{\delta o_j}{\delta net_j}$$

$$= -\frac{\delta E}{\delta o_j} \times o_j (1 - o_j)$$

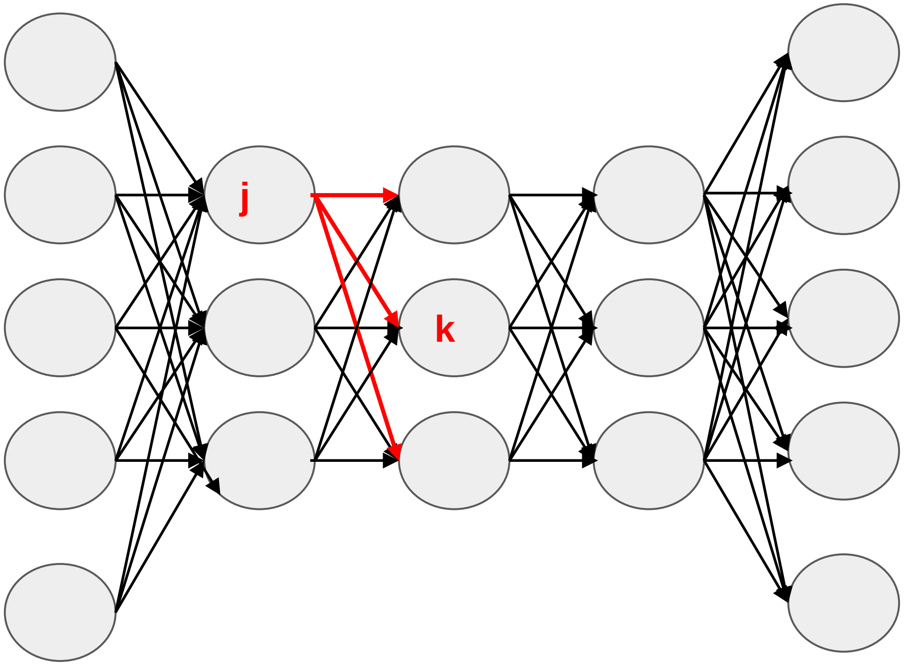
$$= -\sum_{k \in \text{next layer}} \left(\frac{\delta E}{\delta net_k} \times \frac{\delta net_k}{\delta o_j}\right) \times o_j (1 - o_j)$$

$$\text{Hence, } \delta_j = -\sum_{k \in \text{next layer}} (-\delta_k \times w_{kj}) \times o_j (1 - o_j)$$

This recursion can give rise to vanishing and exploding Gradient problem

$$= \sum_{k \in \text{next layer}} (w_{kj} \delta_k) o_j (1 - o_j)$$

Back-propagation- for hidden layers: Impact on net input on a neuron



- O_j affects the net input coming to all the neurons in next layer

General Backpropagation Rule

- General weight updating rule:

$$\Delta w_{ji} = \eta \delta j o_i$$

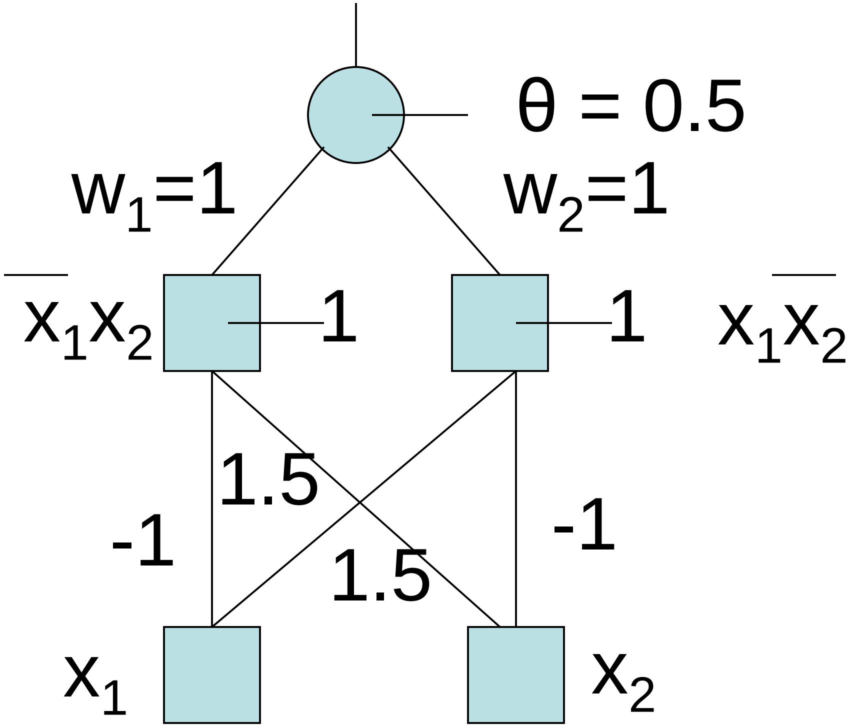
- Where

$$\delta_j = (t_j - o_j) o_j (1 - o_j) \quad \text{for outermost layer}$$

$$= \sum_{k \in \text{next layer}} (w_{kj} \delta_k) o_j (1 - o_j) o_i \text{ for hidden layers}$$

How does it work?

Input propagation forward and error propagation backward (e.g. XOR)



Optional Assignment

- Implement your OWN BP on XOR
- Observe what the hidden layer neurons compute

An application in Medical Domain

Expert System for Skin Diseases Diagnosis

- Bumpiness and scaliness of skin
- Mostly for symptom gathering and for developing diagnosis skills
- Not replacing doctor’s diagnosis

Architecture of the FF NN

- 96-20-10
- 96 input neurons, 20 hidden layer neurons, 10 output neurons
- Inputs: skin disease symptoms and their parameters
  - *Location, distribution, shape, arrangement, pattern, number of lesions, presence of an active norder, amount of scale, elevation of papuls, color, altered pigmentation, itching, pustules, lymphadenopathy, palmer thickening, results of microscopic examination, presence of herald pathc, result of dermatology test called KOH*

Output

- 10 neurons indicative of the diseases:
  - *psoriasis, pityriasis rubra pilaris, lichen planus, pityriasis rosea, tinea versicolor, dermatophytosis, cutaneous T-cell lymphoma, secondery syphilis, chronic contact dermatitis, soberrheic dermatitis*

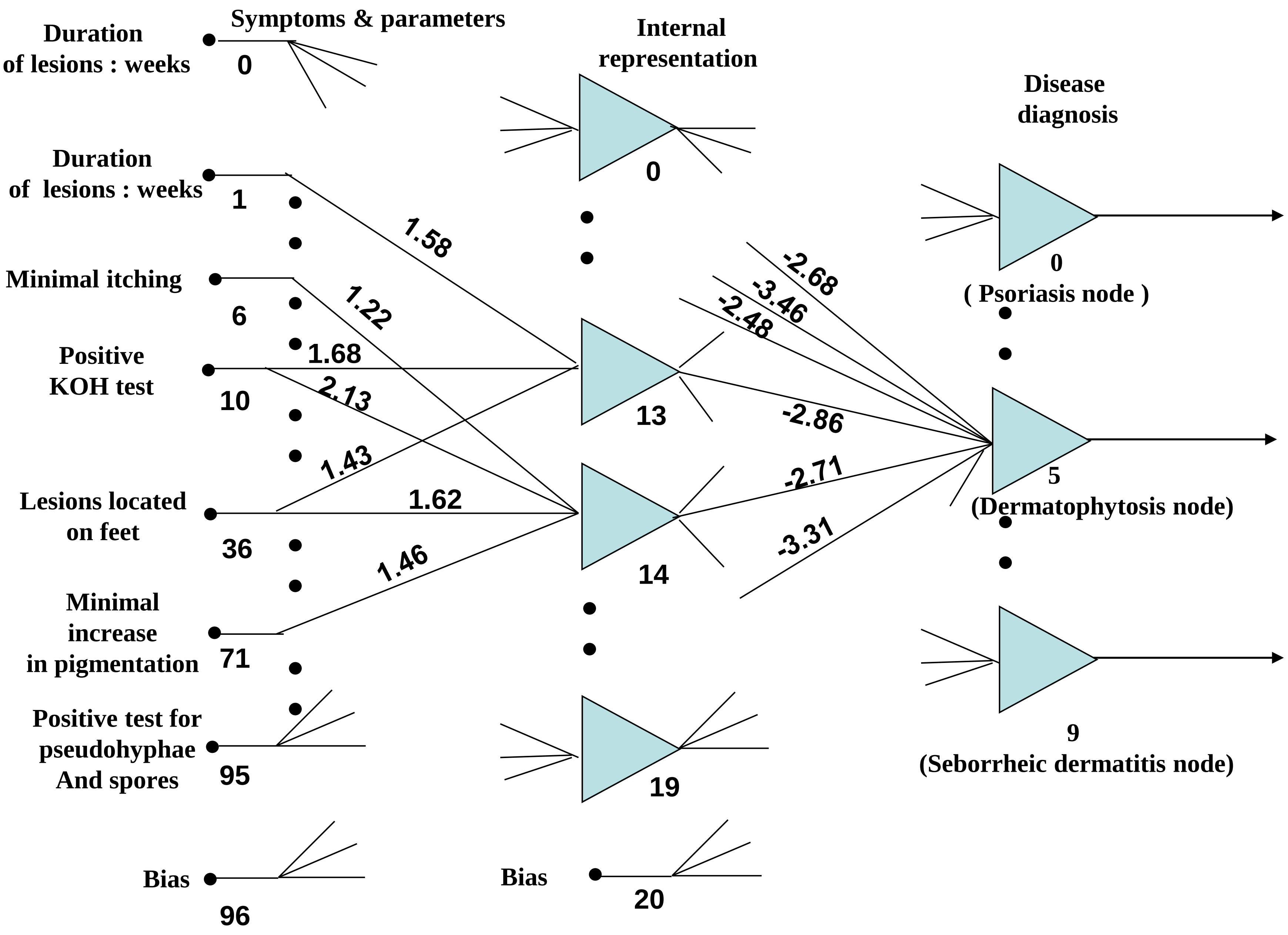


Figure : Explanation of dermatophytosis diagnosis using the DESKNET expert system.

Training data

- Input specs of 10 model diseases from 250 patients
- 0.5 is some specific symptom value is not known
- Trained using standard error backpropagation algorithm

Testing

- Previously unused symptom and disease data of 99 patients
- Result:
- Correct diagnosis achieved for 70% of papulosquamous group skin diseases
- Success rate above 80% for the remaining diseases except for psoriasis
- psoriasis diagnosed correctly only in 30% of the cases
- Psoriasis resembles other diseases within the papulosquamous group of diseases, and is somewhat difficult even for specialists to recognise.

Explanation capability

- Rule based systems reveal the explicit path of reasoning through the textual statements
- Connectionist expert systems reach conclusions through complex, non linear and simultaneous interaction of many units
- Analysing the effect of a single input or a single group of inputs would be difficult and would yield incorrect results

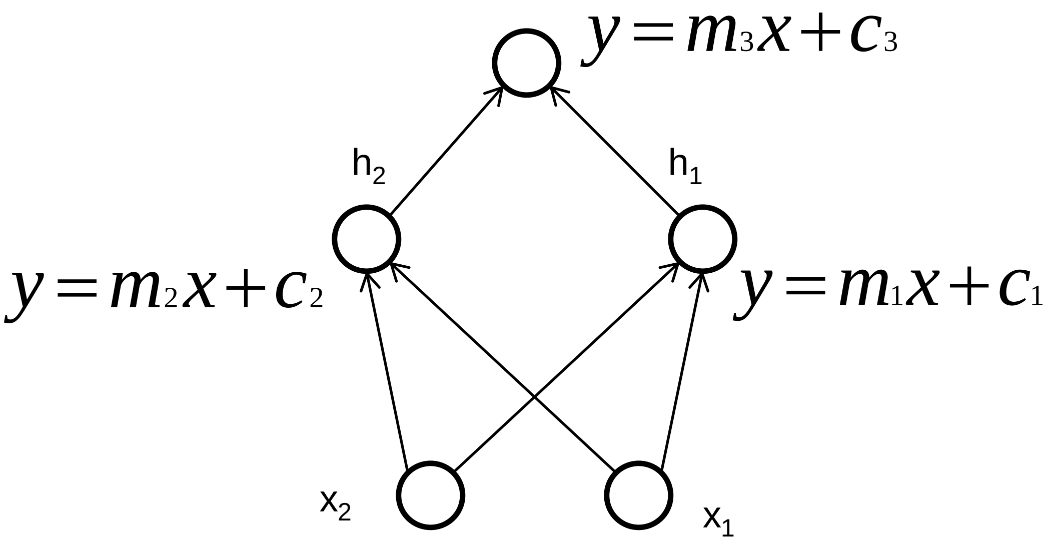
Explanation contd.

- The hidden layer re-represents the data
- Outputs of hidden neurons are neither symtoms nor decisions

Discussion

- Symptoms and parameters contributing to the diagnosis found from the n/w
- Standard deviation, mean and other tests of significance used to arrive at the importance of contributing parameters
- The n/w acts as apprentice to the expert

Can Linear Neurons Work?



$$h_1=m_1(w_1x_1+w_2x_2)+c_1$$

$$h_1=m_2(w_1x_1+w_2x_2)+c_2$$

$$Out=(w_5h_1+w_6h_2)+c_3$$

$$=k_1x_1+k_2x_2+k_3$$

Note: The whole structure shown in earlier slide is reducible to a single neuron with given behavior

$$Out = k_1 x_1 + k_2 x_2 + k_3$$

Claim: A neuron with linear I-O behavior can’t compute X-OR.

Proof: Considering all possible cases:

[assuming 0.1 and 0.9 as the lower and upper thresholds]

For (0,0), Zero class:

$$m(w_1.0 + w_2.0 - \theta) + c < 0.1$$
$$\Rightarrow c - m.\theta < 0.1$$

For (0,1), One class:

$$m(w_1.1 + w_2.0 - \theta) + c > 0.9$$
$$\Rightarrow m.w_1 - m.\theta + c > 0.9$$

For (1,0), One class: $m.w_2 - m.\theta + c > 0.9$

For (1,1), Zero class: $m.w_1 - m_2.\theta + c < 0.1$

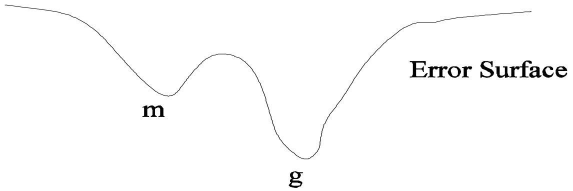
These equations are inconsistent. Hence X-OR can’t be computed.

Observations:

1. A linear neuron can’t compute X-OR.
2. A multilayer FFN with linear neurons is collapsible to a single linear neuron, hence **no a additional power due to hidden layer.**
3. Non-linearity is essential for power.

Local Minima

Due to the Greedy nature of BP, it can get stuck in local minimum *m* and will never be able to reach the global minimum *g* as the error can only decrease by weight change.



m- local minima, g- global minima

Figure- Getting Stuck in local minimum

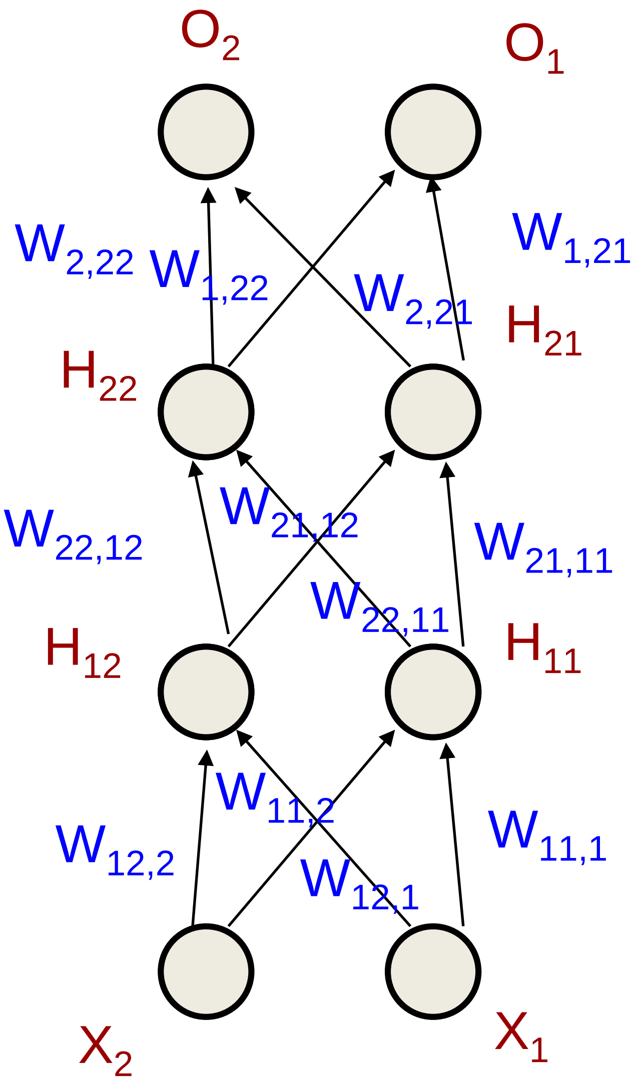
Momentum factor

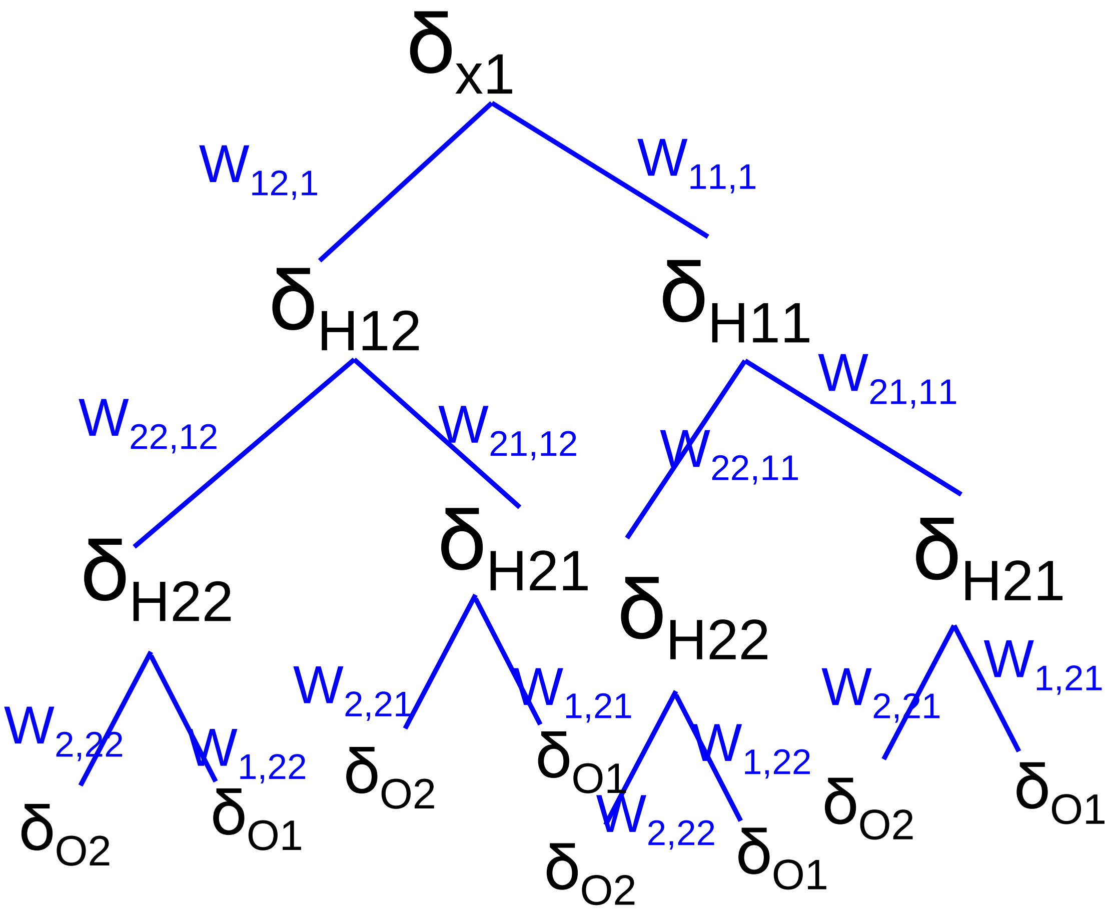
1. Introduce momentum factor.

$$(\Delta w_{ji})_{nth-iteration} = \eta \delta_j O_i + \beta (\Delta w_{ji})_{(n-1)th-iteration}$$

- Accelerates the movement out of the trough.
- Dampens oscillation inside the trough.
- Choosing β : If β is large, we may jump over the minimum.

Vanishing/Exploding Gradient





$$\delta_{x1}=W_{11,1}\delta_{H11}+W_{12,1}\delta_{H12}$$

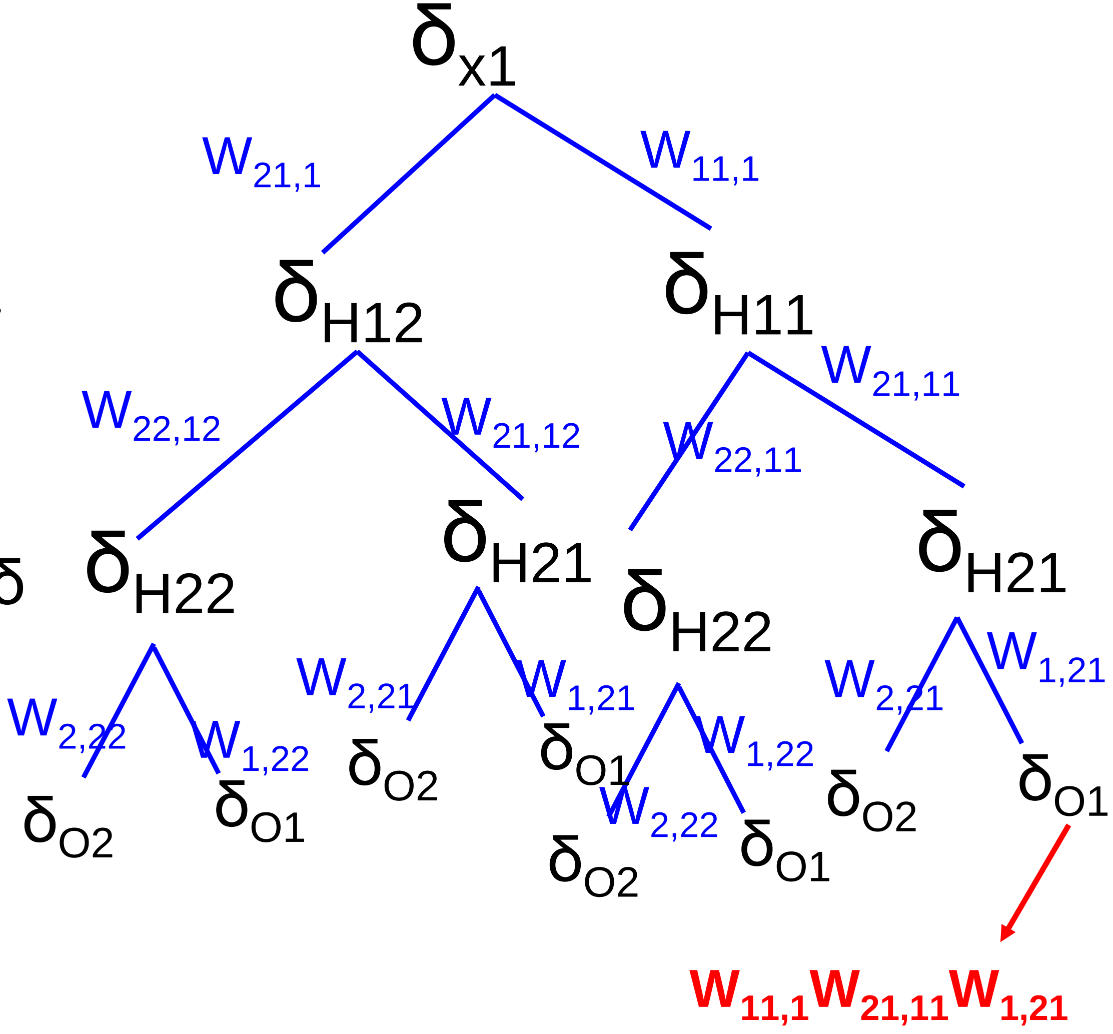
Vanishing/Exploding Gradient

$\delta_{x1}=W_{11,1}\delta_{H11}+W_{21,1}\delta_{H12}$ [2 terms]

$=W_{11,1}(W_{21,11}\delta_{H21}+W_{22,11}\delta_{H22})+W_{21,1}(W_{21,12}\delta_{H21} + W_{22,12}\delta_{H22})$ [4 terms]

$= W_{11,1}W_{21,11}\delta_{H21}+W_{11,1}W_{22,21}\delta_{H22}+W_{21,1}W_{21,12}\delta_{H21}+ W_{21,1}W_{22,12}\delta_{H22}$

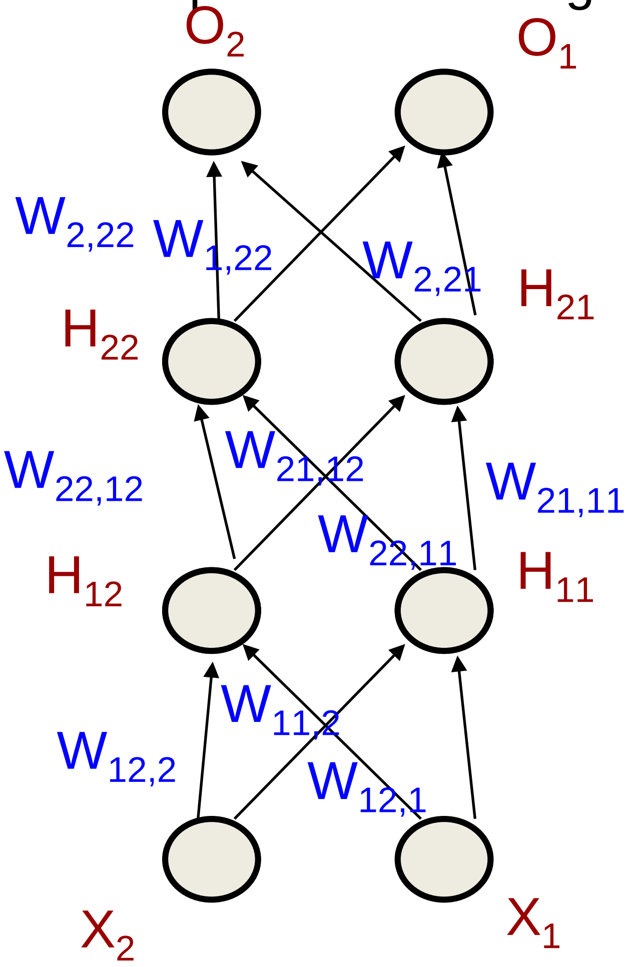
= (4 terms with δ_{o1}) + (4 terms with δ_{o2}; one term shown for the leftmost leaf's weight)

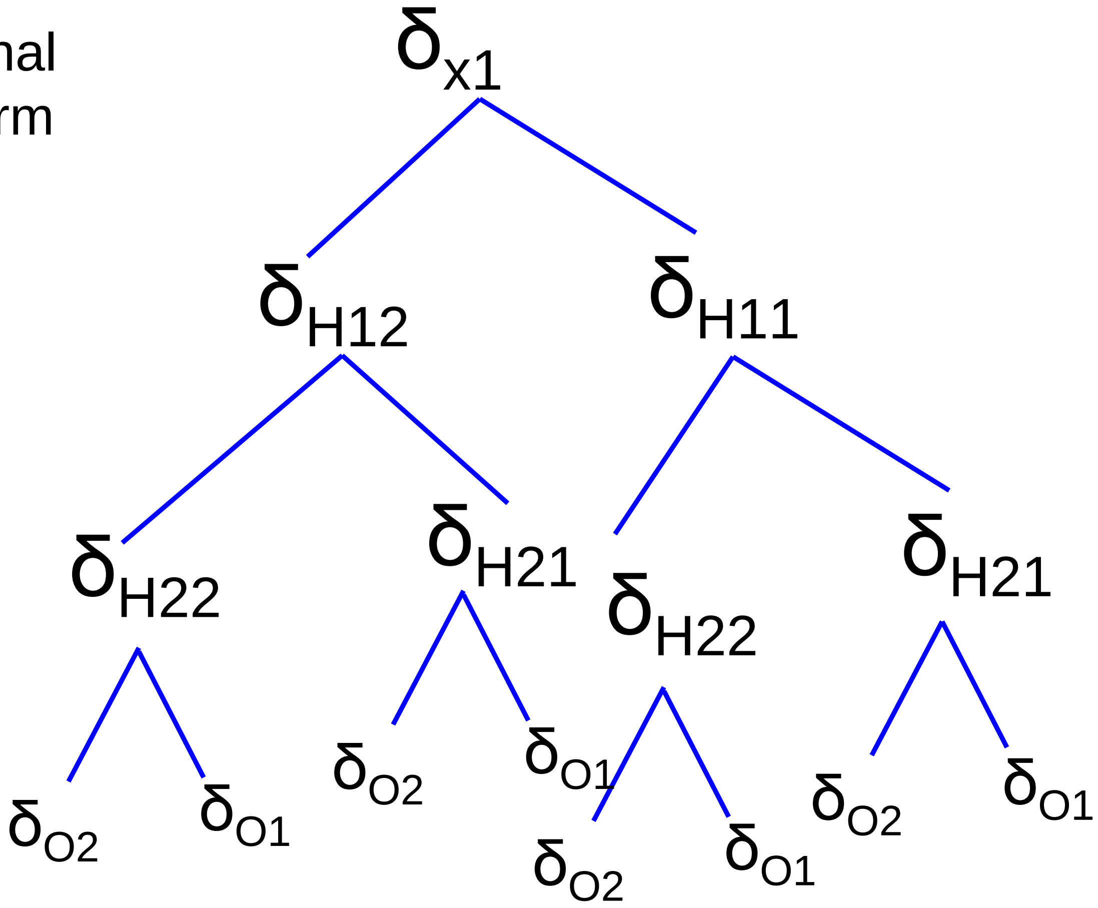


$\mathbf{W_{11,1}W_{21,11}W_{1,21}}$

Vanishing/Exploding Gradient

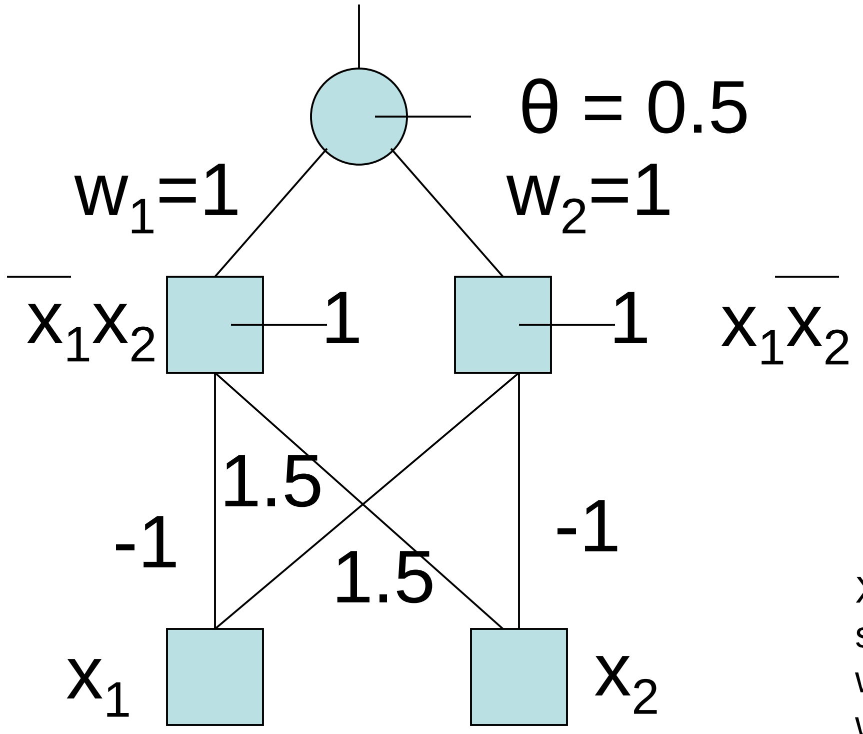
With ‘*B*’ as branching factor and
‘*L*’ as number of levels,
There will be B^L terms in the final
Expansion of δ_{x1}. Also each term
Will be product of L weights





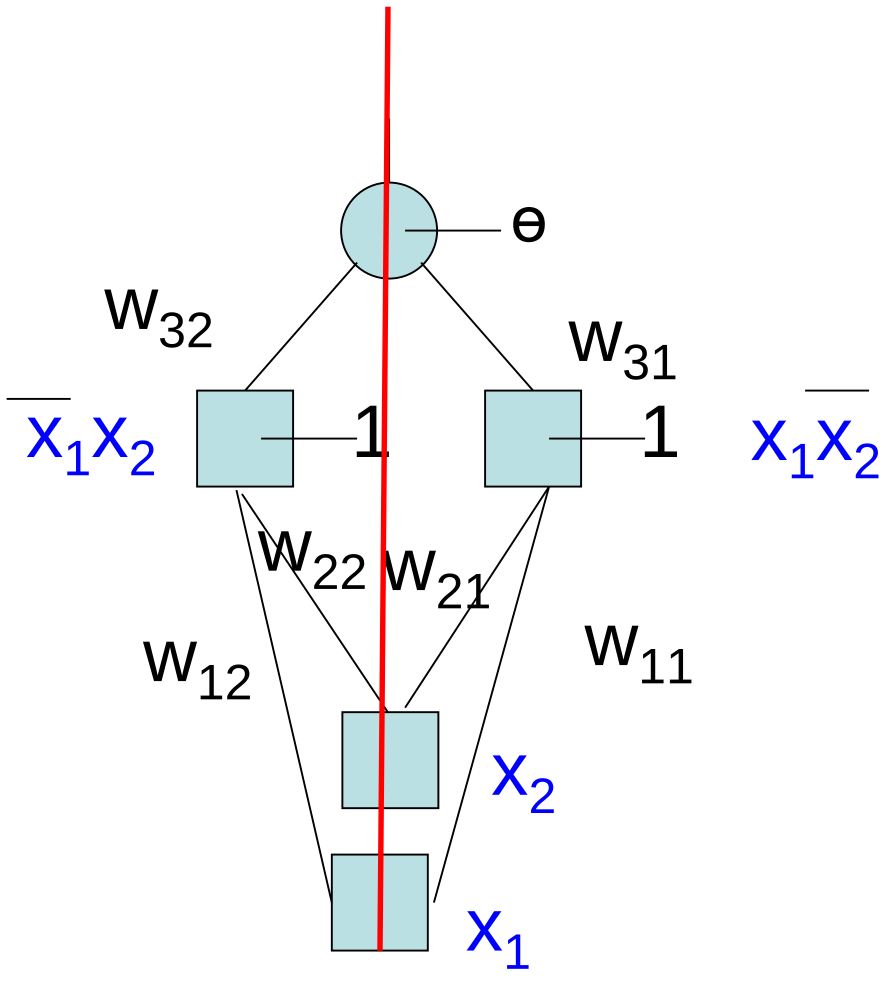
Symmetry breaking

- If mapping demands different weights, but we start with the same weights everywhere, then BP will never converge.



XOR n/w: if we s started with identical weight everywhere, BP will not converge

Symmetry breaking: understanding with proper diagram



Symmetry
About
The red
Line should
Be broken